# DO NOW

Find the derivative of $f(x) = 2x^3(3x - 5)$

$f(x) = 6x^4 - 10x^3$

$\boxed{f'(x) = 24x^3 - 30x^2}$

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## 3.3 Product and Quotient Rules

**The Product Rule**:

$\frac{d}{dx}[f(x)g(x)] = f(x)g'(x) + g(x)f'(x)$

$1^{st}\left(\text{derivative of } 2^{nd}\right) + 2^{nd}\left(\text{derivative of } 1^{st}\right)$

Extension:

$\frac{d}{dx}[f(x)g(x)h(x)] =$

$f'(x)g(x)h(x) + f(x)g'(x)h(x) + f(x)g(x)h'(x)$

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Prove the product rule.

$\frac{d}{dx}[f(x)g(x)] = \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$

$\lim_{\Delta x \to 0} \frac{f(x+\Delta x)g(x+\Delta x) - f(x)g(x)}{\Delta x}$

$\lim_{\Delta x \to 0} \frac{f(x+\Delta x)g(x+\Delta x) - f(x+\Delta x)g(x) + f(x+\Delta x)g(x) - f(x)g(x)}{\Delta x}$

$\lim_{\Delta x \to 0} \left[\frac{f(x+\Delta x)g(x+\Delta x) - f(x+\Delta x)g(x)}{\Delta x} + \frac{f(x+\Delta x)g(x) - f(x)g(x)}{\Delta x}\right]$

$\lim_{\Delta x \to 0} \left[f(x+\Delta x)\frac{g(x+\Delta x) - g(x)}{\Delta x} + g(x)\frac{f(x+\Delta x) - f(x)}{\Delta x}\right]$

$f(x)g'(x) + g(x)f'(x)$

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Examples: Find the derivatives of each function.

1. $f(x) = 2x^3(3x - 5)$

$f'(x) = 1^{st}\left(\text{deriv of } 2^{nd}\right) + 2^{nd}\left(\text{deriv of } 1^{st}\right)$

$f'(x) = 2x^3(3) + (3x-5)(6x^2)$

$f'(x) = 6x^3 + 18x^3 - 30x^2$

$\boxed{f'(x) = 24x^3 - 30x^2}$

2. $h(t) = (2x - 4x^2)(3 - x)$

$h'(t) = (2x - 4x^2)(-1) + (3-x)(2-8x)$

$h'(t) = -2x + 4x^2 + 6 - 24x - 2x + 8x^2$

$\boxed{h'(t) = 12x^2 - 28x + 6}$

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3. $g(x) = xe^x$

$1^{st}$ function

$2^{nd}$ function

$g'(x) = x(e^x) + e^x(1)$

$g'(x) = xe^x + e^x$

ok

$g'(x) = e^x(x+1)$

4. $y = 2x\cos x - 2\sin x$

$y' = 2x(-\sin x) + (\cos x)(2) - 2\cos x$

$y' = -2x\sin x + 2\cos x - 2\cos x$

$\boxed{y' = -2x\sin x}$

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**The Quotient Rule**:

$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$

$\frac{\text{denom.}\left(\text{derivative of num.}\right) - \text{num.}\left(\text{derivative of denom.}\right)}{(\text{denom.})^2}$

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Prove the quotient rule.

$$\lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x}$$

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right]$$

$$\lim_{\Delta x \to 0} \frac{\frac{f(x+\Delta x)}{g(x+\Delta x)} - \frac{f(x)}{g(x)}}{\Delta x}$$

*Multiply top & bottom by $g(x)g(x+\Delta x)$

$$\lim_{\Delta x \to 0} \frac{f(x+\Delta x)g(x) - f(x)g(x+\Delta x)}{(\Delta x)g(x)g(x+\Delta x)}$$

$$\lim_{\Delta x \to 0} \frac{f(x+\Delta x)g(x) - f(x)g(x) + f(x)g(x) - f(x)g(x+\Delta x)}{(\Delta x)g(x)g(x+\Delta x)}$$

$$\lim_{\Delta x \to 0} \frac{\frac{f(x+\Delta x)g(x) - f(x)g(x)}{\Delta x} + \frac{f(x)g(x) - f(x)g(x+\Delta x)}{\Delta x}}{g(x)g(x+\Delta x)}$$



$$\lim_{\Delta x \to 0} \frac{\frac{f(x+\Delta x)g(x) - f(x)g(x)}{\Delta x} + \frac{f(x)g(x) - f(x)g(x+\Delta x)}{\Delta x}}{g(x)g(x+\Delta x)}$$

$$\lim_{\Delta x \to 0} \frac{g(x)\frac{f(x+\Delta x) - f(x)}{\Delta x} - f(x)\frac{g(x+\Delta x) - g(x)}{\Delta x}}{g(x)g(x+\Delta x)}$$

$$\frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$$



Examples:

5. $f(x) = \frac{3x^2 - 5x}{2x^3}$

$$f'(x) = \frac{2x^3(6x-5) - (3x^2-5x)(6x^2)}{(2x^3)^2}$$

$$f'(x) = \frac{12x^4 - 10x^3 - 18x^4 + 30x^3}{4x^6}$$

$$f'(x) = \frac{-6x^4 + 20x^3}{4x^6}$$

$$f'(x) = \frac{2x^3(-3x+10)}{4x^6}$$

$$\boxed{f'(x) = \frac{-3x+10}{2x^3}}$$



6. $y = \frac{2x^3}{x - 5}$

$$y' = \frac{(x-5)(6x^2) - 2x^3(1)}{(x-5)^2}$$

$$y' = \frac{6x^3 - 30x^2 - 2x^3}{(x-5)^2}$$

$$y' = \frac{4x^3 - 30x^2}{(x-5)^2}$$

$$\boxed{y' = \frac{2x^2(2x-15)}{(x-5)^2}}$$



**Rules for simplifying:**

1. Eliminate all negative exponents.
2. Combine like terms.
3. Reduce all fractions.
4. Leave written as a single fraction if possible.
5. Leave denominator in factored form.



# HOMEWORK

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